# Energy correlation and asymmetry of secondary leptons originating in 


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#### Abstract

We study the energy correlation of charged leptons produced in the decay of a heavy Higgs particle $H \rightarrow t \bar{t} \rightarrow b l^{+} \nu_{l} \bar{b} l^{-} \bar{\nu}_{l}$ and $H \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l}$. The possible influence of $C P$-violation in the $H t \bar{t}$ and $H W^{+} W^{-}$vertices on the energy spectrum of the secondary leptons is analyzed. The energy distribution of the charged leptons in the decay $H \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l}$ is sensitive to the $C P$-parity of the Higgs particle and yields a simple criterion for distinguishing scalar Higgs from pseudoscalar Higgs.


## 1 Introduction

We wish to report results on the energy spectrum and energy correlation of charged leptons produced in the reactions

$$
\begin{align*}
H & \rightarrow t \bar{t} \rightarrow b l^{+} \nu_{l} \bar{b} l^{-} \bar{\nu}_{l}  \tag{1}\\
H & \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l} . \tag{2}
\end{align*}
$$

The above decays represent interesting leptonic signals of a heavy Higgs particle, that can be used to test the structure of Higgs couplings to fermions and gauge bosons [1]. (Note that the reaction (2), in the standard model, is about 27 times more frequent than the "gold-plated" reaction $H \rightarrow Z Z \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}$). We carry out the analysis in a general framework in which the couplings of the $H$ to $t \bar{t}$ and to $W^{+} W^{-}$are given by:

$$
\begin{array}{rll}
H t \bar{t} & : & i\left(a+i b \gamma_{5}\right), \\
H W^{+} W^{-} & : & i 2 m_{W}^{2} \sqrt{G_{F} \sqrt{2}}\left(B g_{\mu \nu}+\frac{D}{m_{W}^{2}} \varepsilon_{\mu \nu \rho \sigma} p_{W^{+}}^{\rho} p_{W^{-}}^{\sigma}\right) . \tag{4}
\end{array}
$$

Here $p_{W^{+}}$and $p_{W^{-}}$are the 4 -momenta of the $W$-bosons. The terms proportional to $b$ and $D$ may arise as primary or induced effects in a generalized Higgs framework. Simultaneous presence of $a$ and $b$ or $B$ and $D$ is $C P$-violating [2]. Results will be obtained for the energy correlation of the two charged leptons in the $H$ rest frame. A special result is a simple criterion for distinguishing a scalar Higgs from a pseudoscalar Higgs particle on the basis of the energy spectrum of any single charged lepton in $H \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l}$.

## $2 H \rightarrow t \bar{t}$

The vertex $H t \bar{t}$ (Eq. (3)) gives rise to the following differential decay rate for $H(P) \rightarrow t\left(p_{t}, s_{+}\right) \bar{t}\left(p_{\bar{t}}, s_{-}\right):$

$$
\begin{align*}
\frac{\mathrm{d} \Gamma}{\mathrm{~d} \Omega_{t}}\left(s_{+}, s_{-}\right)= & \frac{\beta_{t}}{64 \pi^{2} m_{H}}\left\{\left(|a|^{2}+|b|^{2}\right)\left(\frac{m_{H}^{2}}{2}-m_{t}^{2}+m_{t}^{2} s_{+} s_{-}\right)\right. \\
& +\left(|a|^{2}-|b|^{2}\right)\left(P s_{+} P s_{-}-\frac{m_{H}^{2}}{2} s_{+} s_{-}+m_{t}^{2} s_{+} s_{-}-m_{t}^{2}\right) \\
& \left.-\operatorname{Re}\left(a b^{*}\right) \varepsilon\left(P, Q, s_{+}, s_{-}\right)-2 \operatorname{Im}\left(a b^{*}\right) m_{t} P\left(s_{+}+s_{-}\right)\right\}, \tag{5}
\end{align*}
$$

where $P \equiv p_{t}+p_{\bar{t}}, Q=p_{t}-p_{\bar{t}}$, and $s_{+}$and $s_{-}$denote the polarization vectors of $t$ and $\bar{t}$, respectively. $\beta_{t}=\sqrt{1-4 m_{t}^{2} / m_{H}^{2}}$ is the velocity of the top quarks in the Higgs rest frame. The symbol $\varepsilon(a, b, c, d)$ means $\varepsilon_{\mu \nu \rho \sigma} a^{\mu} b^{\nu} c^{\rho} d^{\sigma}$ with $\varepsilon_{0123}=+1$. The terms proportional to $\operatorname{Re}\left(a b^{*}\right)$ and $\operatorname{Im}\left(a b^{*}\right)$ represent the $C P$-violating part of the differential decay rate.

Using the method of Kawasaki, Shirafuji and Tsai [3], the differential decay rate $\frac{\mathrm{d} \Gamma}{\mathrm{d} \Omega_{t}}\left(s_{+}, s_{-}\right)$yields the following normalized energy correlation of the charged leptons produced in the decay $H \rightarrow t \bar{t} \rightarrow l^{+} l^{-}+\cdots$ [F 1]:

$$
\begin{align*}
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} x \mathrm{~d} x^{\prime}}\left(H \rightarrow t \bar{t} \rightarrow l^{+} l^{-}+\cdots\right)= & f(x) f\left(x^{\prime}\right)-\frac{1}{\beta_{t}^{2}} g(x) g\left(x^{\prime}\right) \\
& +\frac{2 \operatorname{Im}\left(a b^{*}\right)}{|a|^{2} \beta_{t}^{2}+|b|^{2}}\left[f\left(x^{\prime}\right) g(x)-f(x) g\left(x^{\prime}\right)\right] \tag{6}
\end{align*}
$$

where $x$ and $x^{\prime}$ are the reduced energies

$$
\begin{equation*}
x=\frac{2 E\left(l^{+}\right)}{m_{t}} \sqrt{\frac{1-\beta_{t}}{1+\beta_{t}}} \quad, \quad x^{\prime}=\frac{2 E\left(l^{-}\right)}{m_{t}} \sqrt{\frac{1-\beta_{t}}{1+\beta_{t}}}, \tag{7}
\end{equation*}
$$

$E\left(l^{+}\right)$and $E\left(l^{-}\right)$being the energies of the final leptons $l^{+}$and $l^{-}$in the Higgs rest frame. $x$ and $x^{\prime}$ are bounded by

$$
\begin{equation*}
\frac{m_{W}^{2}}{m_{t}^{2}} \frac{1-\beta_{t}}{1+\beta_{t}} \leq x, x^{\prime} \leq 1, \tag{8}
\end{equation*}
$$

assuming the narrow width approximation for the $W$-bosons in the top decay. The functions $f$ and $g$ are defined as follows (see [四):

1. $\frac{m_{W}^{2}}{m_{t}^{2}} \geq \frac{1-\beta_{t}}{1+\beta_{t}}$

$$
\begin{aligned}
& f(x)=\frac{3}{2 W} \frac{1+\beta_{t}}{\beta_{t}} \begin{cases}-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}}+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2} & : I_{1} \\
1-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}} & : I_{2} \\
1-2 x+x^{2} & : I_{3}\end{cases} \\
& g(x)=\frac{3}{W} \frac{\left(1+\beta_{t}\right)^{2}}{\beta_{t}} \begin{cases}-x \frac{m_{W}^{2}}{m_{t}^{2}}+x^{2} \frac{1+\beta_{t}}{1-\beta_{t}}+x \ln \frac{m_{W}^{2}}{m_{t}^{2}}-x \ln \left(x \frac{1+\beta_{t}}{1-\beta_{t}}\right) \\
+\frac{1 / 2}{1+\beta_{t}}\left[-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}}+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2}\right] & : I_{1} \\
x-x \frac{m_{W}^{2}}{m_{t}^{2}}+x \ln \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{1 / 2}{1+\beta_{t}}\left[1-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}}\right] & : I_{2} \\
x-x^{2}+x \ln x+\frac{1 / 2}{1+\beta_{t}}\left[1-2 x+x^{2}\right] & : I_{3}\end{cases}
\end{aligned}
$$

where the intervals $I_{i}$ are given by:

$$
\begin{aligned}
I_{1}: \frac{m_{W}^{2}}{m_{t}^{2}} \frac{1-\beta_{t}}{1+\beta_{t}} & \leq x \leq \frac{1-\beta_{t}}{1+\beta_{t}}, \\
I_{2}: \frac{1-\beta_{t}}{1+\beta_{t}} & \leq x \leq \frac{m_{W}^{2}}{m_{t}^{2}} \\
I_{3}: & \frac{m_{W}^{2}}{m_{t}^{2}}
\end{aligned}
$$

2. $\frac{m_{W}^{2}}{m_{t}^{2}} \leq \frac{1-\beta_{t}}{1+\beta_{t}}$

$$
f(x)=\frac{3}{2 W} \frac{1+\beta_{t}}{\beta_{t}} \begin{cases}-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}}+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2} & : I_{4} \\ -2 x+x^{2}+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2} & : I_{5} \\ 1-2 x+x^{2} & : I_{6}\end{cases}
$$

$$
g(x)=\frac{3}{W} \frac{\left(1+\beta_{t}\right)^{2}}{\beta_{t}} \begin{cases}-x \frac{m_{W}^{2}}{m_{t}^{2}}+x^{2} \frac{1+\beta_{t}}{1-\beta_{t}}+x \ln \frac{m_{W}^{2}}{m_{t}^{2}}-x \ln \left(x \frac{1+\beta_{t}}{1-\beta_{t}}\right) \\ +\frac{1 / 2}{1+\beta_{t}}\left[-2 \frac{m_{W}^{2}}{m_{t}^{2}}+\frac{m_{W}^{4}}{m_{t}^{4}}+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2}\right] & : I_{4} \\ -x^{2}+x^{2} \frac{1+\beta_{t}}{1-\beta_{t}}+x \ln \frac{1-\beta_{t}}{1+\beta_{t}}+\frac{1 / 2}{1+\beta_{t}}\left[-2 x+x^{2}\right. \\ \left.+2 x \frac{1+\beta_{t}}{1-\beta_{t}}-x^{2}\left(\frac{1+\beta_{t}}{1-\beta_{t}}\right)^{2}\right] & : I_{5} \\ x-x^{2}+x \ln x+\frac{1 / 2}{1+\beta_{t}}\left[1-2 x+x^{2}\right] & : I_{6}\end{cases}
$$

with the intervals $I_{i}$ :

$$
\begin{aligned}
& I_{4}: \frac{m_{W}^{2}}{m_{t}^{2}} \frac{1-\beta_{t}}{1+\beta_{t}} \leq x \leq \frac{m_{W}^{2}}{m_{t}^{2}}, \\
& I_{5}: \frac{m_{W}^{2}}{m_{t}^{2}} \leq x \leq \frac{1-\beta_{t}}{1+\beta_{t}}, \\
& I_{6}: \frac{1-\beta_{t}}{1+\beta_{t}} \leq x \leq 1,
\end{aligned}
$$

and

$$
\begin{equation*}
W=\left(1-\frac{m_{W}^{2}}{m_{t}^{2}}\right)^{2}\left(1+2 \frac{m_{W}^{2}}{m_{t}^{2}}\right) . \tag{9}
\end{equation*}
$$

The normalizations of $f$ and $g$ are

$$
\begin{array}{r}
\int f(x) d x=1 \\
\int g(x) d x=0 . \tag{10}
\end{array}
$$

The functions $f$ and $g$ represent the spin-independent and spin-dependent parts of the lepton spectrum in $t$-decay. Eq. (6) can also be written as

$$
\begin{equation*}
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} x \mathrm{~d} x^{\prime}}\left(H \rightarrow t \bar{t} \rightarrow l^{+} l^{-}+\cdots\right)=S_{t}\left(x, x^{\prime}\right)+\Delta A_{t}\left(x, x^{\prime}\right) \tag{11}
\end{equation*}
$$

where

$$
\begin{align*}
S_{t}\left(x, x^{\prime}\right) & =f(x) f\left(x^{\prime}\right)-\frac{1}{\beta_{t}^{2}} g(x) g\left(x^{\prime}\right) \\
A_{t}\left(x, x^{\prime}\right) & =f\left(x^{\prime}\right) g(x)-f(x) g\left(x^{\prime}\right) \\
\Delta & =\frac{2 \operatorname{Im}\left(a b^{*}\right)}{|a|^{2} \beta_{t}^{2}+|b|^{2}} \tag{12}
\end{align*}
$$

$S_{t}\left(x, x^{\prime}\right)$ and $A_{t}\left(x, x^{\prime}\right)$ represent the symmetric and antisymmetric part of the energy correlation. These are plotted in Figs. (1a) and (1b).

The symmetric ( $C P$-conserving) part of the two-dimensional distribution $\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} x \mathrm{~d} x^{\prime}}$ does not depend on the coupling constants $a$ and $b$. This means that in the $C P-$ conserving limit the energy correlation of secondary leptons arising from $H \rightarrow t \bar{t}$ is independent of the $C P$-parity of the decaying Higgs particle.

Integration over $x$ or $x^{\prime}$ yields the single lepton energy spectra

$$
\begin{equation*}
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} x}\left(H \rightarrow t \bar{t} \rightarrow l^{ \pm}+\cdots\right)=f(x) \pm \Delta g(x) \tag{13}
\end{equation*}
$$

Eq. (13) agrees with the energy spectrum obtained by Chang and Keung [[]] using a different method. The single energy spectra are plotted in Fig. 2. The parameter $\Delta$ is calculated within a 2 -Higgs Doublet Model in Refs. 5 , 6.

## $3 H \rightarrow W^{+} W^{-}$

The differential decay rate for the reaction $H(P) \rightarrow W^{+} W^{-} \rightarrow l^{+}\left(q_{1}\right) \nu_{l}\left(q_{2}\right) l^{-}\left(q_{3}\right) \bar{\nu}_{l}\left(q_{4}\right)$, arising from the $H W^{+} W^{-}$vertex given in Eq. (4), is

$$
\begin{equation*}
\mathrm{d}^{8} \Gamma=8 \sqrt{2} \frac{G_{F}}{m_{H}} D_{W}\left[|B|^{2} \mathcal{S}+\frac{|D|^{2}}{m_{W}^{4}} \mathcal{P}+\frac{\operatorname{Re}\left(B D^{*}\right)}{m_{W}^{2}} \mathcal{Q}-\frac{\operatorname{Im}\left(B D^{*}\right)}{m_{W}^{2}} \mathcal{R}\right] \cdot \mathrm{d} \text { Lips } \tag{14}
\end{equation*}
$$

The Lorentz invariant phase space is given by

$$
\begin{equation*}
\mathrm{d} \text { Lips }=(2 \pi)^{4} \delta^{(4)}\left(P-q_{1}-q_{2}-q_{3}-q_{4}\right) \prod_{i=1}^{4} \frac{\mathrm{~d}^{3} q_{i}}{(2 \pi)^{3} 2 q_{i}^{0}} . \tag{15}
\end{equation*}
$$

In the massless fermion approximation,

$$
\begin{align*}
\mathcal{S}= & \left(q_{2} \cdot q_{3}\right)\left(q_{1} \cdot q_{4}\right), \\
\mathcal{P}= & -\left\{\left(q_{2} \cdot q_{3}\right)\left(q_{1} \cdot q_{4}\right)-\left(q_{2} \cdot q_{4}\right)\left(q_{1} \cdot q_{3}\right)\right\}^{2} \\
& +\frac{m_{W}^{4}}{4}\left\{\left(q_{2} \cdot q_{3}\right)^{2}+\left(q_{1} \cdot q_{4}\right)^{2}+2\left(q_{2} \cdot q_{4}\right)\left(q_{1} \cdot q_{3}\right)-\frac{m_{W}^{4}}{4}\right\}, \\
\mathcal{Q}= & \varepsilon\left(q_{1}, q_{2}, q_{3}, q_{4}\right)\left\{\left(q_{2} \cdot q_{3}\right)+\left(q_{1} \cdot q_{4}\right)\right\} \\
\mathcal{R}= & \left\{\left(q_{2} \cdot q_{3}\right)-\left(q_{1} \cdot q_{4}\right)\right\}\left\{\frac{m_{W}^{4}}{4}+\left(q_{2} \cdot q_{3}\right)\left(q_{1} \cdot q_{4}\right)-\left(q_{2} \cdot q_{4}\right)\left(q_{1} \cdot q_{3}\right)\right\} \tag{16}
\end{align*}
$$

while $D_{W}$ is the propagator factor

$$
\begin{equation*}
D_{W}=m_{W}^{4} \prod_{j=1}^{2} \frac{g^{2}}{\left(s_{j}-m_{W}^{2}\right)^{2}+m_{W}^{2} \Gamma_{W}^{2}} \tag{17}
\end{equation*}
$$

with $s_{1}=\left(q_{1}+q_{2}\right)^{2}, s_{2}=\left(q_{3}+q_{4}\right)^{2}$. In the narrow width approximation, the total decay rate is given by

$$
\begin{align*}
\Gamma\left(H \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l}\right)= & \frac{g^{6} m_{H}^{3} \beta_{W}}{9 \cdot 2^{16} \pi^{3} \Gamma_{W}^{2}}\{ \\
& \left.|B|^{2}\left(3-2 \beta_{W}^{2}+3 \beta_{W}^{4}\right)+8|D|^{2} \beta_{W}^{2}\right\} \tag{18}
\end{align*}
$$

in agreement with the result of Osland and Skjold [7].
We now introduce scaled energy variables in the $H$ rest frame:

$$
\begin{equation*}
y=\frac{4 E\left(l^{+}\right)}{m_{H}} \quad, \quad y^{\prime}=\frac{4 E\left(l^{-}\right)}{m_{H}} \tag{19}
\end{equation*}
$$

which are bounded by

$$
\begin{equation*}
1-\beta_{W} \leq y, y^{\prime} \leq 1+\beta_{W} \tag{20}
\end{equation*}
$$

where $\beta_{W}=\sqrt{1-4 m_{W}^{2} / m_{H}^{2}}$. The two-dimensional spectrum in the variables $y$ and $y^{\prime}$ is then given by

$$
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} y \mathrm{~d} y^{\prime}}\left(H \rightarrow W^{+} W^{-} \rightarrow l^{+} l^{-}+\cdots\right)=\frac{1}{|B|^{2}\left(3-2 \beta_{W}^{2}+3 \beta_{W}^{4}\right)+8|D|^{2} \beta_{W}^{2}} \cdot \frac{9}{32 \beta_{W}^{6}}
$$

$$
\begin{align*}
& \left\{| B | ^ { 2 } \left[\left(3+2 \beta_{W}^{2}+3 \beta_{W}^{4}\right)\left((y-1)^{2}-\beta_{W}^{2}\right)\left(\left(y^{\prime}-1\right)^{2}-\beta_{W}^{2}\right)\right.\right. \\
& \left.+2 \beta_{W}^{2}\left(1-\beta_{W}^{2}\right)^{2}\left(y-y^{\prime}\right)^{2}\right] \\
& +4 \beta_{W}^{2}|D|^{2}\left[\left((y-1)^{2}+\beta_{W}^{2}\right)\left(\left(y^{\prime}-1\right)^{2}+\beta_{W}^{2}\right)-4 \beta_{W}^{2}(y-1)\left(y^{\prime}-1\right)\right] \\
& \left.+8 \beta_{W}^{2} \operatorname{Im}\left(B D^{*}\right)\left(1-\beta_{W}^{2}\right)\left[\beta_{W}^{2}-(y-1)\left(y^{\prime}-1\right)\right]\left(y-y^{\prime}\right)\right\} . \tag{21}
\end{align*}
$$

Neglecting terms proportional to $|D|^{2}$, the correlation can be written as

$$
\begin{align*}
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} y \mathrm{~d} y^{\prime}}\left(H \rightarrow W^{+} W^{-} \rightarrow l^{+} l^{-}+\cdots\right)=S_{W}\left(y, y^{\prime}\right) & +\frac{\operatorname{Im}\left(B D^{*}\right)}{|B|^{2}} A_{W}\left(y, y^{\prime}\right) \\
& +O\left(|D|^{2} /|B|^{2}\right) \tag{22}
\end{align*}
$$

Here $S_{W}\left(y, y^{\prime}\right)$ and $A_{W}\left(y, y^{\prime}\right)$ represent the symmetric and antisymmetric parts of the energy correlation of the charged leptons, the latter being multiplied by the $C P$-violating coefficient $\operatorname{Im}\left(B D^{*}\right) /|B|^{2}$. These functions are plotted in Figs. (3a) and (3b).

There is an interesting difference in the energy characteristics of the leptons emanating from $H \rightarrow W^{+} W^{-} \rightarrow l^{+} \nu_{l} l^{-} \bar{\nu}_{l}$, dependent on whether $H$ is a scalar $\left(0^{+}\right)$ or pseudoscalar $\left(0^{-}\right)$particle. The correlated energy spectrum of the $l^{+} l^{-}$pair can be derived from Eq. (21) by taking $D=0$ (scalar case) or $B=0$ (pseudoscalar case), with the result

$$
\begin{align*}
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma\left(0^{+}\right)}{\mathrm{d} y \mathrm{~d} y^{\prime}}=S_{W}\left(y, y^{\prime}\right)= & \frac{9}{32 \beta_{W}^{6}} \frac{1}{3-2 \beta_{W}^{2}+3 \beta_{W}^{4}}\left[2 \beta_{W}^{2}\left(1-\beta_{W}^{2}\right)^{2}\left(y-y^{\prime}\right)^{2}\right. \\
& \left.+\left(3+2 \beta_{W}^{2}+3 \beta_{W}^{4}\right)\left((y-1)^{2}-\beta_{W}^{2}\right)\left(\left(y^{\prime}-1\right)^{2}-\beta_{W}^{2}\right)\right],  \tag{23}\\
\frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma\left(0^{-}\right)}{\mathrm{d} y \mathrm{~d} y^{\prime}}=P_{W}\left(y, y^{\prime}\right)= & \frac{9}{64 \beta_{W}^{6}}\left[\left((y-1)^{2}+\beta_{W}^{2}\right)\left(\left(y^{\prime}-1\right)^{2}+\beta_{W}^{2}\right)\right. \\
& \left.-4 \beta_{W}^{2}(y-1)\left(y^{\prime}-1\right)\right] . \tag{24}
\end{align*}
$$

These two functions are strikingly different, as shown in Figs. (3a) and (3c). This difference persists even if we consider the energy spectrum of a single lepton. Inte-
grating Eqs. $(23,24)$ over $y^{\prime}$, we have

$$
\begin{align*}
& \frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma\left(0^{+}\right)}{\mathrm{d} y}=\frac{3}{2 \beta_{W}} \frac{1+\beta_{W}^{4}-2(y-1)^{2}}{3-2 \beta_{W}^{2}+3 \beta_{W}^{4}}  \tag{25}\\
& \frac{1}{\Gamma} \frac{\mathrm{~d} \Gamma\left(0^{-}\right)}{\mathrm{d} y}=\frac{3}{8 \beta_{W}^{3}}\left(\beta_{W}^{2}+(y-1)^{2}\right) \tag{26}
\end{align*}
$$

These distributions are clearly quite distinct (Fig. 4) and provide a simple criterion for distinguishing $0^{+}$and $0^{-}$objects decaying into $W^{+} W^{-}$pairs. Indeed, the single lepton spectra (Eqs. (25), (26)) are also valid for the inclusive process $H \rightarrow W^{+} W^{-} \rightarrow l^{ \pm} X$, where only one of the $W$-bosons decays leptonically. The difference in the lepton energy spectrum for the $0^{+}$and $0^{-}$cases is intimately related to the different helicity structure of the $W$-bosons produced in the two cases [8]. It should be stressed that the correlations and spectra given above (Eqs. (21)-(26)) refer directly to energies measured in the $H$ rest frame, and do not require reconstruction of the decay planes of $W^{+}$and $W^{-}$. In this respect, the present criterion provides a useful alternative to other criteria that have recently been proposed in the literature [8, [9]. Finally, we note that the energy spectrum in the $0^{+}$case agrees with that obtained by Chang and Keung [5], after correction of a minor typographical error [F 2].

One of us (T.A.) acknowledges a stipend from the NRW Graduiertenförderungsprogramm. This research has been supported by the BMFT (German Ministry of Research and Technology).

## Footnotes

[F 1 ] Some of the essential steps in the procedure of Ref. [3] can be found in Ref. (1)
[F 2 ] Eq. (15) of Ref. [5] should read

$$
\frac{1}{N} \frac{\mathrm{~d} N}{\mathrm{~d} x\left(l^{ \pm}\right)}=\left(\frac{\left(1+\beta_{W}^{2}\right)^{2}}{3-2 \beta_{W}^{2}+3 \beta_{W}^{4}}\right) \frac{3\left[\beta_{W}^{2}-(1-x)^{2}\right]}{4 \beta_{W}^{3}}+\sum_{s=-1,+1} \cdots
$$

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## Figure Captions

Fig. 1. $C P$-conserving (a) and $C P$-violating (b) part of the normalized energy correlation in the decay $H \rightarrow t \bar{t} \rightarrow l^{+} l^{-}+\cdots$ for $m_{H}=400 \mathrm{GeV}$ and $m_{t}=150$ GeV .

Fig. 2. Single particle energy spectra of $l^{+}$(dotted curve) and $l^{-}$(full curve) in the decay $H \rightarrow t \bar{t}$ for $\Delta=0.1, m_{H}=400 \mathrm{GeV}$ and $m_{t}=150 \mathrm{GeV}$.

Fig. 3. $C P$-conserving (a) and $C P$-violating (b) part of the normalized energy correlation in the decay $H \rightarrow W^{+} W^{-} \rightarrow l^{+} l^{-}+\cdots$ for $m_{H}=300 \mathrm{GeV}$. Fig. 3(c) shows the normalized energy correlation for the decay of a pseudoscalar Higgs $H \rightarrow W^{+} W^{-} \rightarrow l^{+} l^{-}+\cdots$ for $m_{H}=300 \mathrm{GeV}$.

Fig. 4. Energy distribution of a single lepton in the decay $H \rightarrow W^{+} W^{-} \rightarrow l^{ \pm}+\cdots$ for $m_{H}=300 \mathrm{GeV}$. The full curve represents the scalar case and the dotted curve shows the pseudoscalar case.


Fig. 2.


Fig. 4.

$$
S_{w}\left(y, y^{\prime}\right)
$$



Fig. 3(a).

$$
A_{w}\left(y, y^{\prime}\right)
$$



$$
P_{w}\left(y, y^{\prime}\right)
$$



$$
S_{t}\left(X, X^{\prime}\right)
$$



Fig. 1 (a).


Fig. 1 (b).

