Energy correlation and asymmetry of secondary leptons originating in $H \to t\bar{t}$ and $H \to W^+W^-$

T. Arens, U.D.J. Gieseler and L.M. Sehgal

III. Physikalisches Institut (A), RWTH Aachen,

D-52074 Aachen, Germany

Abstract

We study the energy correlation of charged leptons produced in the decay of a heavy Higgs particle $H \to t\bar{t} \to bl^+\nu_l\bar{b}l^-\bar{\nu}_l$ and $H \to W^+W^- \to l^+\nu_l l^-\bar{\nu}_l$. The possible influence of CP-violation in the $Ht\bar{t}$ and HW^+W^- vertices on the energy spectrum of the secondary leptons is analyzed. The energy distribution of the charged leptons in the decay $H \to W^+W^- \to l^+\nu_l l^-\bar{\nu}_l$ is sensitive to the CP-parity of the Higgs particle and yields a simple criterion for distinguishing scalar Higgs from pseudoscalar Higgs.

1 Introduction

We wish to report results on the energy spectrum and energy correlation of charged leptons produced in the reactions

$$H \rightarrow t\bar{t} \rightarrow bl^+ \nu_l \bar{b} l^- \bar{\nu}_l, \tag{1}$$

$$H \rightarrow W^+ W^- \rightarrow l^+ \nu_l l^- \bar{\nu}_l. \tag{2}$$

The above decays represent interesting leptonic signals of a heavy Higgs particle, that can be used to test the structure of Higgs couplings to fermions and gauge bosons [1]. (Note that the reaction (2), in the standard model, is about 27 times more frequent than the "gold–plated" reaction $H \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$). We carry out the analysis in a general framework in which the couplings of the H to $t\bar{t}$ and to W^+W^- are given by:

$$Ht\bar{t} : i(a+ib\gamma_5), \tag{3}$$

$$HW^{+}W^{-} : i2m_{W}^{2}\sqrt{G_{F}\sqrt{2}}(Bg_{\mu\nu} + \frac{D}{m_{W}^{2}}\varepsilon_{\mu\nu\rho\sigma}p_{W^{+}}^{\rho}p_{W^{-}}^{\sigma}).$$
(4)

Here p_{W^+} and p_{W^-} are the 4-momenta of the W-bosons. The terms proportional to b and D may arise as primary or induced effects in a generalized Higgs framework. Simultaneous presence of a and b or B and D is CP-violating [2]. Results will be obtained for the energy correlation of the two charged leptons in the H rest frame. A special result is a simple criterion for distinguishing a scalar Higgs from a pseudoscalar Higgs particle on the basis of the energy spectrum of any single charged lepton in $H \to W^+W^- \to l^+\nu_l l^-\bar{\nu}_l$.

$\mathbf{2} \quad H \to t\bar{t}$

The vertex $Ht\bar{t}$ (Eq. (3)) gives rise to the following differential decay rate for $H(P) \rightarrow t(p_t, s_+)\bar{t}(p_{\bar{t}}, s_-)$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_t}(s_+, s_-) = \frac{\beta_t}{64\pi^2 m_H} \Big\{ (|a|^2 + |b|^2) (\frac{m_H^2}{2} - m_t^2 + m_t^2 s_+ s_-) \\
+ (|a|^2 - |b|^2) (Ps_+ Ps_- - \frac{m_H^2}{2} s_+ s_- + m_t^2 s_+ s_- - m_t^2) \\
- \mathrm{Re}(ab^*) \varepsilon(P, Q, s_+, s_-) - 2\mathrm{Im}(ab^*) m_t P(s_+ + s_-) \Big\},$$
(5)

where $P \equiv p_t + p_{\bar{t}}$, $Q = p_t - p_{\bar{t}}$, and s_+ and s_- denote the polarization vectors of t and \bar{t} , respectively. $\beta_t = \sqrt{1 - 4m_t^2/m_H^2}$ is the velocity of the top quarks in the Higgs rest frame. The symbol $\varepsilon(a, b, c, d)$ means $\varepsilon_{\mu\nu\rho\sigma}a^{\mu}b^{\nu}c^{\rho}d^{\sigma}$ with $\varepsilon_{0123} = +1$. The terms proportional to $\operatorname{Re}(ab^*)$ and $\operatorname{Im}(ab^*)$ represent the CP-violating part of the differential decay rate.

Using the method of Kawasaki, Shirafuji and Tsai [3], the differential decay rate $\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega_t}(s_+, s_-)$ yields the following normalized energy correlation of the charged leptons produced in the decay $H \to t\bar{t} \to l^+l^- + \cdots$ [F 1]:

$$\frac{1}{\Gamma} \frac{d\Gamma}{dxdx'} (H \to t\bar{t} \to l^+ l^- + \cdots) = f(x)f(x') - \frac{1}{\beta_t^2}g(x)g(x') \\
+ \frac{2\mathrm{Im}(ab^*)}{|a|^2\beta_t^2 + |b|^2} \Big[f(x')g(x) - f(x)g(x')\Big], \quad (6)$$

where x and x' are the reduced energies

$$x = \frac{2E(l^{+})}{m_t} \sqrt{\frac{1-\beta_t}{1+\beta_t}} , \qquad x' = \frac{2E(l^{-})}{m_t} \sqrt{\frac{1-\beta_t}{1+\beta_t}}, \qquad (7)$$

 $E(l^+)$ and $E(l^-)$ being the energies of the final leptons l^+ and l^- in the Higgs rest frame. x and x' are bounded by

$$\frac{m_W^2}{m_t^2} \frac{1 - \beta_t}{1 + \beta_t} \le x, x' \le 1,$$
(8)

assuming the narrow width approximation for the W-bosons in the top decay. The functions f and g are defined as follows (see [4]):

$$\begin{split} 1. \ \ \frac{m_W^2}{m_t^2} &\geq \frac{1-\beta_t}{1+\beta_t} \\ f(x) &= \frac{3}{2W} \frac{1+\beta_t}{\beta_t} \begin{cases} -2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta_t}{1-\beta_t} - x^2(\frac{1+\beta_t}{1-\beta_t})^2 &: I_1 \\ 1-2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} &: I_2 \\ 1-2x+x^2 &: I_3 \end{cases} \\ f(x) &= \frac{3}{W} \frac{(1+\beta_t)^2}{\beta_t} \end{cases} \begin{cases} -x\frac{m_W^2}{m_t^2} + x^2\frac{1+\beta_t}{1-\beta_t} + x\ln\frac{m_W^2}{m_t^2} - x\ln(x\frac{1+\beta_t}{1-\beta_t}) \\ +\frac{1/2}{1+\beta_t} [-2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta_t}{1-\beta_t} - x^2(\frac{1+\beta_t}{1-\beta_t})^2] &: I_1 \\ x - x\frac{m_W^2}{m_t^2} + x\ln\frac{m_W^2}{m_t^2} + \frac{1/2}{1+\beta_t} [1-2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4}] &: I_2 \\ x - x^2 + x\ln x + \frac{1/2}{1+\beta_t} [1-2x+x^2] &: I_3 \end{cases} \end{split}$$

where the intervals I_i are given by:

$$I_{1} : \frac{m_{W}^{2}}{m_{t}^{2}} \frac{1 - \beta_{t}}{1 + \beta_{t}} \leq x \leq \frac{1 - \beta_{t}}{1 + \beta_{t}},$$

$$I_{2} : \frac{1 - \beta_{t}}{1 + \beta_{t}} \leq x \leq \frac{m_{W}^{2}}{m_{t}^{2}},$$

$$I_{3} : \frac{m_{W}^{2}}{m_{t}^{2}} \leq x \leq 1.$$

$$\begin{aligned} 2. \ \ \frac{m_W^2}{m_t^2} &\leq \frac{1-\beta_t}{1+\beta_t} \\ f(x) &= \frac{3}{2W} \frac{1+\beta_t}{\beta_t} \begin{cases} -2\frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x\frac{1+\beta_t}{1-\beta_t} - x^2(\frac{1+\beta_t}{1-\beta_t})^2 & :I_4 \\ -2x + x^2 + 2x\frac{1+\beta_t}{1-\beta_t} - x^2(\frac{1+\beta_t}{1-\beta_t})^2 & :I_5 \\ 1 - 2x + x^2 & :I_6 \end{cases} \end{aligned}$$

$$g(x) = \frac{3}{W} \frac{(1+\beta_t)^2}{\beta_t} \begin{cases} -x \frac{m_W^2}{m_t^2} + x^2 \frac{1+\beta_t}{1-\beta_t} + x \ln \frac{m_W^2}{m_t^2} - x \ln(x \frac{1+\beta_t}{1-\beta_t}) \\ + \frac{1/2}{1+\beta_t} [-2 \frac{m_W^2}{m_t^2} + \frac{m_W^4}{m_t^4} + 2x \frac{1+\beta_t}{1-\beta_t} - x^2 (\frac{1+\beta_t}{1-\beta_t})^2] & : I_4 \\ -x^2 + x^2 \frac{1+\beta_t}{1-\beta_t} + x \ln \frac{1-\beta_t}{1+\beta_t} + \frac{1/2}{1+\beta_t} [-2x + x^2 + 2x \frac{1+\beta_t}{1-\beta_t} - x^2 (\frac{1+\beta_t}{1-\beta_t})^2] & : I_5 \\ + 2x \frac{1+\beta_t}{1-\beta_t} - x^2 (\frac{1+\beta_t}{1-\beta_t})^2] & : I_5 \\ x - x^2 + x \ln x + \frac{1/2}{1+\beta_t} [1-2x + x^2] & : I_6 \end{cases}$$

with the intervals I_i :

$$I_{4} : \frac{m_{W}^{2}}{m_{t}^{2}} \frac{1 - \beta_{t}}{1 + \beta_{t}} \leq x \leq \frac{m_{W}^{2}}{m_{t}^{2}},$$

$$I_{5} : \frac{m_{W}^{2}}{m_{t}^{2}} \leq x \leq \frac{1 - \beta_{t}}{1 + \beta_{t}},$$

$$I_{6} : \frac{1 - \beta_{t}}{1 + \beta_{t}} \leq x \leq 1,$$

and

$$W = (1 - \frac{m_W^2}{m_t^2})^2 (1 + 2\frac{m_W^2}{m_t^2}).$$
(9)

The normalizations of f and g are

$$\int f(x)dx = 1,$$

$$\int g(x)dx = 0.$$
 (10)

The functions f and g represent the spin-independent and spin-dependent parts of the lepton spectrum in t-decay. Eq. (6) can also be written as

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}x\mathrm{d}x'}(H\to t\bar{t}\to l^+l^-+\cdots) = S_t(x,x') + \Delta A_t(x,x'),\tag{11}$$

where

$$S_{t}(x, x') = f(x)f(x') - \frac{1}{\beta_{t}^{2}}g(x)g(x'),$$

$$A_{t}(x, x') = f(x')g(x) - f(x)g(x'),$$

$$\Delta = \frac{2\text{Im}(ab^{*})}{|a|^{2}\beta_{t}^{2} + |b|^{2}}.$$
(12)

 $S_t(x, x')$ and $A_t(x, x')$ represent the symmetric and antisymmetric part of the energy correlation. These are plotted in Figs. (1a) and (1b).

The symmetric (*CP*-conserving) part of the two-dimensional distribution $\frac{1}{\Gamma} \frac{d\Gamma}{dxdx'}$ does not depend on the coupling constants *a* and *b*. This means that in the *CP*conserving limit the energy correlation of secondary leptons arising from $H \to t\bar{t}$ is independent of the *CP*-parity of the decaying Higgs particle.

Integration over x or x' yields the single lepton energy spectra

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}x}(H \to t\bar{t} \to l^{\pm} + \cdots) = f(x) \pm \Delta g(x).$$
(13)

Eq. (13) agrees with the energy spectrum obtained by Chang and Keung [5] using a different method. The single energy spectra are plotted in Fig. 2. The parameter Δ is calculated within a 2–Higgs Doublet Model in Refs. [5, 6].

$3 \quad H \to W^+ W^-$

The differential decay rate for the reaction $H(P) \to W^+W^- \to l^+(q_1)\nu_l(q_2)l^-(q_3)\bar{\nu}_l(q_4)$, arising from the HW^+W^- vertex given in Eq. (4), is

$$d^{8}\Gamma = 8\sqrt{2}\frac{G_{F}}{m_{H}}D_{W}\left[|B|^{2}\mathcal{S} + \frac{|D|^{2}}{m_{W}^{4}}\mathcal{P} + \frac{\operatorname{Re}(BD^{*})}{m_{W}^{2}}\mathcal{Q} - \frac{\operatorname{Im}(BD^{*})}{m_{W}^{2}}\mathcal{R}\right] \cdot dLips.$$
(14)

The Lorentz invariant phase space is given by

$$dLips = (2\pi)^4 \delta^{(4)} (P - q_1 - q_2 - q_3 - q_4) \prod_{i=1}^4 \frac{d^3 q_i}{(2\pi)^3 2q_i^0}.$$
 (15)

In the massless fermion approximation,

$$S = (q_2 \cdot q_3)(q_1 \cdot q_4),$$

$$\mathcal{P} = -\left\{ (q_2 \cdot q_3)(q_1 \cdot q_4) - (q_2 \cdot q_4)(q_1 \cdot q_3) \right\}^2 + \frac{m_W^4}{4} \left\{ (q_2 \cdot q_3)^2 + (q_1 \cdot q_4)^2 + 2(q_2 \cdot q_4)(q_1 \cdot q_3) - \frac{m_W^4}{4} \right\},$$

$$\mathcal{Q} = \varepsilon(q_1, q_2, q_3, q_4) \left\{ (q_2 \cdot q_3) + (q_1 \cdot q_4) \right\},$$

$$\mathcal{R} = \left\{ (q_2 \cdot q_3) - (q_1 \cdot q_4) \right\} \left\{ \frac{m_W^4}{4} + (q_2 \cdot q_3)(q_1 \cdot q_4) - (q_2 \cdot q_4)(q_1 \cdot q_3) \right\}, \quad (16)$$

while D_W is the propagator factor

$$D_W = m_W^4 \prod_{j=1}^2 \frac{g^2}{(s_j - m_W^2)^2 + m_W^2 \Gamma_W^2},$$
(17)

with $s_1 = (q_1 + q_2)^2$, $s_2 = (q_3 + q_4)^2$. In the narrow width approximation, the total decay rate is given by

$$\Gamma(H \to W^+ W^- \to l^+ \nu_l l^- \bar{\nu}_l) = \frac{g^6 m_H^3 \beta_W}{9 \cdot 2^{16} \pi^3 \Gamma_W^2} \Big\{ |B|^2 (3 - 2\beta_W^2 + 3\beta_W^4) + 8|D|^2 \beta_W^2 \Big\}, \quad (18)$$

in agreement with the result of Osland and Skjold [7].

We now introduce scaled energy variables in the H rest frame:

$$y = \frac{4E(l^+)}{m_H}$$
 , $y' = \frac{4E(l^-)}{m_H}$, (19)

which are bounded by

$$1 - \beta_W \le y, y' \le 1 + \beta_W, \tag{20}$$

where $\beta_W = \sqrt{1 - 4m_W^2/m_H^2}$. The two–dimensional spectrum in the variables y and y' is then given by

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma}{\mathrm{d}y\mathrm{d}y'} (H \to W^+ W^- \to l^+ l^- + \cdots) = \frac{1}{|B|^2 (3 - 2\beta_W^2 + 3\beta_W^4) + 8|D|^2 \beta_W^2} \cdot \frac{9}{32\beta_W^6} \cdot \frac{9}{32\beta_W^6} + \frac{1}{2\beta_W^6} \cdot \frac{1}{2\beta_W^6} + \frac{1}{2\beta_W^6} \cdot \frac{1}{2\beta_W^6} + \frac{1}{2\beta_W^6} \cdot \frac{1}{2\beta_W^6} + \frac{1}{2\beta_W^6} \cdot \frac{1}{2\beta_W^6$$

$$\left\{ |B|^{2} \left[(3 + 2\beta_{W}^{2} + 3\beta_{W}^{4})((y-1)^{2} - \beta_{W}^{2})((y'-1)^{2} - \beta_{W}^{2}) + 2\beta_{W}^{2}(1 - \beta_{W}^{2})^{2}(y-y')^{2} \right] + 4\beta_{W}^{2} |D|^{2} \left[((y-1)^{2} + \beta_{W}^{2})((y'-1)^{2} + \beta_{W}^{2}) - 4\beta_{W}^{2}(y-1)(y'-1) \right] + 8\beta_{W}^{2} \operatorname{Im}(BD^{*})(1 - \beta_{W}^{2}) \left[\beta_{W}^{2} - (y-1)(y'-1) \right] (y-y') \right\}.$$
(21)

Neglecting terms proportional to $|D|^2$, the correlation can be written as

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma}{\mathrm{d}y\mathrm{d}y'} (H \to W^+ W^- \to l^+ l^- + \cdots) = S_W(y, y') + \frac{\mathrm{Im}(BD^*)}{|B|^2} A_W(y, y') + O(|D|^2/|B|^2).$$
(22)

Here $S_W(y, y')$ and $A_W(y, y')$ represent the symmetric and antisymmetric parts of the energy correlation of the charged leptons, the latter being multiplied by the CP-violating coefficient $\text{Im}(BD^*)/|B|^2$. These functions are plotted in Figs. (3a) and (3b).

There is an interesting difference in the energy characteristics of the leptons emanating from $H \to W^+W^- \to l^+\nu_l l^-\bar{\nu}_l$, dependent on whether H is a scalar (0^+) or pseudoscalar (0^-) particle. The correlated energy spectrum of the l^+l^- pair can be derived from Eq. (21) by taking D = 0 (scalar case) or B = 0 (pseudoscalar case), with the result

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma(0^{+})}{\mathrm{d}y\mathrm{d}y'} = S_W(y,y') = \frac{9}{32\beta_W^6} \frac{1}{3-2\beta_W^2+3\beta_W^4} \Big[2\beta_W^2(1-\beta_W^2)^2(y-y')^2 \\ + (3+2\beta_W^2+3\beta_W^4)((y-1)^2-\beta_W^2)((y'-1)^2-\beta_W^2) \Big] (23)$$
$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma(0^{-})}{\mathrm{d}y\mathrm{d}y'} = P_W(y,y') = \frac{9}{64\beta_W^6} \Big[((y-1)^2+\beta_W^2)((y'-1)^2+\beta_W^2) \\ -4\beta_W^2(y-1)(y'-1) \Big].$$
(24)

These two functions are strikingly different, as shown in Figs. (3a) and (3c). This difference persists even if we consider the energy spectrum of a single lepton. Inte-

grating Eqs. (23, 24) over y', we have

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma(0^+)}{\mathrm{d}y} = \frac{3}{2\beta_W} \frac{1 + \beta_W^4 - 2(y-1)^2}{3 - 2\beta_W^2 + 3\beta_W^4},\tag{25}$$

$$\frac{1}{\Gamma} \frac{\mathrm{d}\Gamma(0^{-})}{\mathrm{d}y} = \frac{3}{8\beta_W^3} (\beta_W^2 + (y-1)^2).$$
(26)

These distributions are clearly quite distinct (Fig. 4) and provide a simple criterion for distinguishing 0⁺ and 0⁻ objects decaying into W^+W^- pairs. Indeed, the single lepton spectra (Eqs. (25), (26)) are also valid for the inclusive process $H \to W^+W^- \to l^{\pm}X$, where only one of the W-bosons decays leptonically. The difference in the lepton energy spectrum for the 0⁺ and 0⁻ cases is intimately related to the different helicity structure of the W-bosons produced in the two cases [8]. It should be stressed that the correlations and spectra given above (Eqs. (21)–(26)) refer directly to energies measured in the H rest frame, and do not require reconstruction of the decay planes of W^+ and W^- . In this respect, the present criterion provides a useful alternative to other criteria that have recently been proposed in the literature [8, 9]. Finally, we note that the energy spectrum in the 0⁺ case agrees with that obtained by Chang and Keung [5], after correction of a minor typographical error [F 2].

One of us (T.A.) acknowledges a stipend from the NRW Graduiertenförderungsprogramm. This research has been supported by the BMFT (German Ministry of Research and Technology).

Footnotes

- [F 1] Some of the essential steps in the procedure of Ref. [3] can be found in Ref.[4].
- $[{\rm F}\ 2\]$ Eq. (15) of Ref. [5] should read

$$\frac{1}{N}\frac{\mathrm{d}N}{\mathrm{d}x(l^{\pm})} = \left(\frac{(1+\beta_W^2)^2}{3-2\beta_W^2+3\beta_W^4}\right)\frac{3[\beta_W^2-(1-x)^2]}{4\beta_W^3} + \sum_{s=-1,+1}\cdots$$

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Figure Captions

- Fig. 1. *CP*-conserving (a) and *CP*-violating (b) part of the normalized energy correlation in the decay $H \rightarrow t\bar{t} \rightarrow l^+l^- + \cdots$ for $m_H = 400$ GeV and $m_t = 150$ GeV.
- Fig. 2. Single particle energy spectra of l^+ (dotted curve) and l^- (full curve) in the decay $H \to t\bar{t}$ for $\Delta = 0.1$, $m_H = 400$ GeV and $m_t = 150$ GeV.
- Fig. 3. CP-conserving (a) and CP-violating (b) part of the normalized energy correlation in the decay $H \to W^+W^- \to l^+l^- + \cdots$ for $m_H = 300$ GeV. Fig. 3(c) shows the normalized energy correlation for the decay of a pseudoscalar Higgs $H \to W^+W^- \to l^+l^- + \cdots$ for $m_H = 300$ GeV.
- Fig. 4. Energy distribution of a single lepton in the decay $H \to W^+W^- \to l^{\pm} + \cdots$ for $m_H = 300$ GeV. The full curve represents the scalar case and the dotted curve shows the pseudoscalar case.







Fig. 3(a).



Fig. 3(b).



Fig. 3(c).



Fig. 1(a).



Fig. 1(b).