Proc. 25th Int. Cosmic Ray Conf., Durban (1997) **3**, 13

THE ANGULAR DEPENDENCE OF COMPTONISED RADIATION FROM A DISK

U.D.J. Gieseler and J.G. Kirk

Max-Planck-Institut für Kernphysik, Postfach 10 39 80, D-69029 Heidelberg, Germany

ABSTRACT

We solve the Boltzmann equation describing the comptonisation of low frequency input photons by a thermal distribution of electrons in the Thomson limit, using a semi-analytic method which yields not only the spectral index but also the full spatial and angular dependence of the radiation intensity in a disk geometry. For an optically thin layer of hot plasma, the comptonised radiation is strongly collimated along the disk surface. This has implications for the ratio of the intensity of directly observed nonthermal X-rays to the intensity of radiation reprocessed by a cool surface in models of Seyfert galaxies.

INTRODUCTION

Much of the early work on formation of non thermal continuum spectrum assumed that the optical depth of the scattering cloud was fairly large, and that the photon frequency ν and electron temperature $T_{\rm e}$ were both small: $x \equiv h\nu/m_{\rm e}c^2 \ll 1$, $\Theta \equiv k_{\rm B}T_{\rm e}/m_{\rm e}c^2 \ll 1$. The transport of a photon in both configuration space and in energy space can then be approximated by a Fokker-Planck equation (Sunyaev & Titarchuk 1980). Of particular interest is the generalisation to scattering media of optical depth $\tau \sim 1$, since such conditions are indicated in many applications (e.g., AGN: Haardt et al. 1994; Zdziarski et al. 1995 and galactic black hole candidates: Sunyaev & Trümper 1979; Ebisawa et al. 1996). Titarchuk (1994), pointed out that the angular distribution of comptonised radiation emerging from an optically thin disk forms a 'knife-blade' pattern, collimated almost parallel to the surface of the disk. In addition to analytic work, comptonisation has been investigated using numerical methods (Katz 1976; Sunyaev & Titarchuk 1985; Poutanen & Svensson 1996), in particular the Monte-Carlo simulation technique (Pozdnyakov et al. 1983; Zdziarski 1986; Hua & Titarchuk 1995; Stern et al. 1995a, 1995b).

In a recent paper, Titarchuk & Lyubarskij (1995) (hereafter TL95), have attacked the problem using the Boltzmann equation, without recourse to a Fokker-Planck approximation. We present results found by generalising the method of TL95 by solving the Boltzmann equation without assuming isotropy of either the radiation or the source function (Gieseler & Kirk 1996). The approach we adopt is to formulate the equation determining the power-law index as an integral eigenvalue problem. Our results confirm the accuracy of the formulae presented by TL95 and also give the full spatial and angular dependences of the comptonised radiation.

FORMULATION OF THE PROBLEM

We consider an infinite disk of thickness $2z_0$ containing a uniform, non-degenerate gas of free electrons of temperature $T_{\rm e}$ and number density $n_{\rm e}$. The only process of importance for the transport of photons in this disk is Compton scattering. The optical half-thickness of the disk is defined as $\tau_0 = \sigma_{\rm T} n_{\rm e} z_0$, where $\sigma_{\rm T} = 6.65 \times 10^{-25}\,{\rm cm}^2$ is the Thomson cross-section. Let the spatial coordinate normal to the disk be z, with $z = 0$ on the mid-plane of the disk, and define the optical depth variable $\tau = n_{\rm e}\sigma_{\rm T} z$ which is bounded by $-\tau_0 \leq \tau \leq \tau_0$. We denote the cosine of the angle between the normal to the disk and the direction of a photon after scattering by μ. In the same sense, the direction before a scattering event is denoted by μ_1. Let η be the cosine of the angle between these directions. For an isotropic electron distribution, the phase function, which describes the change in photon direction due to scattering, depends only on η.

We are interested in a situation in which low frequency radiation is injected into the disk, is scattered by the electrons, and forms a power-law spectrum at high frequency, as shown by Shapiro et al. (1976) and Sunyaev & Titarchuk (1980), and seen in numerical studies of Katz (1976). In the power-law regime, there is no source of radiation in the disk, and no radiation which enters from the outside. The time independent, polarisation averaged, equation of transfer for the specific (up-scattered) intensity $I(\nu, \mu, \tau)$ is then given by a special case of the linearised Boltzmann equation (see e.g. Pomraning 1973) and we shall simply refer to it as the Boltzmann equation. The power-law part of the spectrum we describe occurs at frequencies lower than that of the Wien cut-off ($h\nu < k_{\mathrm{B}}T_{\mathrm{e}}$), so that the energy change of the photon due to the recoil of the electron, can be neglected in comparison with the Doppler shift of the photon. We look for a solution of the Boltzmann equation of the form $I(\nu, \mu, \tau) = J(\mu, \tau)\, x^{-\alpha}$. The Boltzmann equation is then

$$\mu \frac{\partial J(\mu, \tau)}{\partial \tau} + J(\mu, \tau) = \frac{1}{4\pi} \int_{-1}^{1} \mathrm{d}\mu_1 \int_0^{2\pi} \mathrm{d}\phi\, R(\eta, \alpha, \Theta)\, J(\mu_1, \tau)\,. \tag{1}$$

The phase function $R(\eta, \alpha, \Theta)$ is a function of the scattering angle, the spectral index and the electron temperature. The electrons are distributed isotropically. They are described by a relativistic Maxwell distribution.

Together with the boundary condition (no particles enter the disk from outside) this equation defines an integral eigenvalue problem. Our aim is to reduce this to an algebraic eigenvalue equation by expanding the intensity $J(\mu, \tau)$ into a polynomial series.

METHOD OF SOLUTION

To reduce the integral eigenvalue problem described above to an algebraic eigenvalue equation, we first expand the phase function in a series of Legendre polynomials (for $\Theta \ll 1$ and $\Theta \gg 1$ separately). The essential step is to expand the specific intensity $J(\mu, \tau)$ into a series of Legendre polynomials for the μ-dependence (to order N) and Chebyshev polynomials for the τ-dependence (to order K). All expansion coefficients can then be represented by a common $(K+1) \cdot (N+1)$ dimensional vector $\mathbf{q}$.

The eigenvalue problem (Eq. 1) becomes with these expansions a set of homogeneous linear equations for the vector $\mathbf{q}$, which is very easy to treat numerically. The set of equations can be written as a matrix equation: $\mathsf{F}(\alpha, \Theta, \tau_0) \cdot \mathbf{q} = 0$. The first step is to calculate the matrix $\mathsf{F}(\alpha, \Theta, \tau_0)$ for given values of τ_0 and Θ. The solution for the spectral index α can be found by requiring the determinant of $\mathsf{F}(\alpha)$ to be zero. For each value of α, which satisfies this condition, the expansion coefficients, and therefore $J(\mu, \tau)$, can be found by solving the equation $\mathsf{F} \cdot \mathbf{q} = 0$, where F is now a known singular square matrix. This matrix can be decomposed using a singular value decomposition routine, which yields the vector null-space. This vector gives immediately the intensity $J(\mu, \tau)$. The resulting values for the spectral index α and the shape of the intensity in μ and τ are described in the next section.

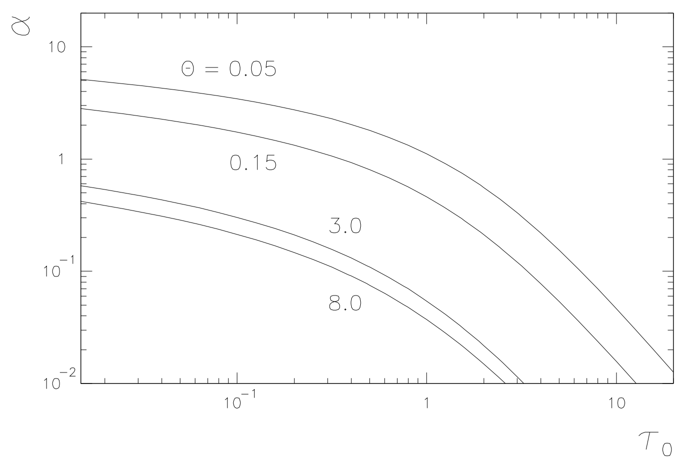

Fig. 1: Spectral index α vs. Thomson optical half thickness τ_0, for non-relativistic and relativistic electron plasma temperatures $\Theta = k_{\mathrm{B}}T_{\mathrm{e}}/m_{\mathrm{e}}c^2$.

SPECTRAL INDEX

Figure 1 shows the spectral index α versus the Thomson optical half thickness τ_0. Results are given for an electron plasma temperature $\Theta = k_B T_e / m_e c^2$ of 0.05 and 0.15 in the non-relativistic regime, corresponding roughly to 25 and 75 keV respectively. In the relativistic regime we choose $\Theta = 3.0$ and $\Theta = 8.0$, corresponding roughly to 1.5 and 4 MeV respectively. These values of α are in good agreement with those given by TL95. These authors assumed an isotropic source function $B(\mu, \tau)$ (right hand side of Eq. 1), which is certainly a good approximation for $\tau_0 \gg 1$. To relax this restriction, at least the first three expansion coefficients of the source function must be taken into account, because of the intrinsic $1 + \eta^2$ dependence of the Thomson scattering kernel. This leads to a discrepancy with the values of TL95 which is at most 10% for smaller values of τ.

ANGULAR DISTRIBUTION

The singular value decomposition of the matrix $\mathbf{F}$ gives $J(\mu, \tau)$ in the form of a polynomial of order K in τ, and of order N in μ (for any set of parameters Θ, τ_0 and the resulting α). Figure 2 shows the intensity $J(\mu, \tau)$ in the half-space $0 \leq \mu \leq 1$ normalised to the intensity in the middle of the disk, parallel to the surface. The boundary condition gives $J(\mu, \tau = -\tau_0) \equiv 0$ for $0 < \mu \leq 1$ (no radiation enters the disk from outside, see dashed line). The dotted line shows the intensity in the middle of the disk ($\tau = 0$), whereas the solid line shows the intensity at the surface, given by $J(\mu, \tau = \tau_0)$. For the optical depth of $2\tau_0 = 0.06$ the radiation in the interval $0 \leq \mu \leq 0.1$, parallel to the disk surface, is a factor of 16 ± 4 more intense than the radiation in the interval $0.9 \leq \mu \leq 1$, perpendicular to the disk. The range of this factor (given as an error) is due to the coupling of electron energy and photon spatial variables. As shown by TL95 the electron energy and photon spatial variables are completely decoupled if the source function is exactly isotropic. However, as discussed in the preceding section, the source function has a weak angular dependence, which leads to a coupling of the electron energy and photon spatial variables. This, in turn, implies that the angular dependence is a function of the plasma temperature. The upper bound of the given factor is valid for $\Theta = 0.01$, whereas the lower bound was calculated for $\Theta = 100$. However, $J(\mu, \tau)$ depends more strongly on the optical depth. The intensity becomes almost isotropic as $2\tau_0$ reaches unity (see Gieseler & Kirk 1996).

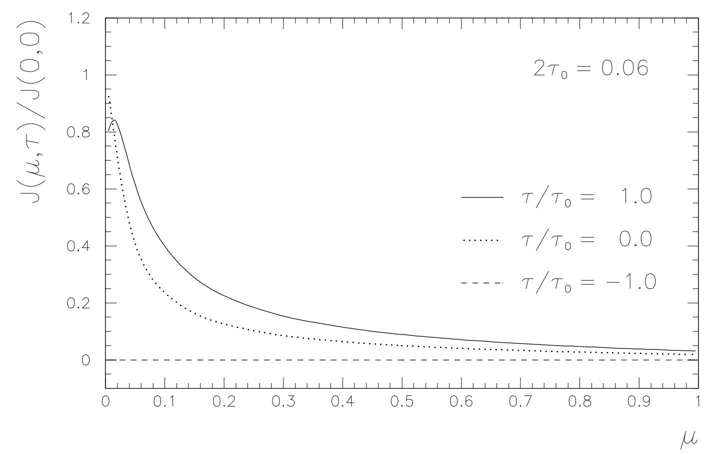

Fig. 2: Angular dependence of the specific intensity $J(\mu, \tau)$ for a Thomson optical thickness of $2\tau_0 = 0.06$. The intensity at the surface of the disk is given by $\tau/\tau_0 = 1.0$.

The reason for the high anisotropy at small optical depth is the following: photons which contribute to the power-law part of the spectrum have to undergo a number of scatterings on electrons to gain the required energy. The energy gain has a maximum for back-scattering of the photons ($\Delta\theta = 180°$). Thus, those photons most effectively boosted in energy and least likely to escape the disk are those which move almost parallel to the surface. This leads to a strong collimation in the disk plane for an optically thin disk.

DISCUSSION

We have presented results of a new, semi-analytic method of obtaining solutions to the comptonisation problem. The results apply to observations of astrophysical objects in the X-ray regime, up to which Thomson scattering is the dominant process for the energy shift of photons, originating from UV, even for the highest temperatures considered here. These photons have to undergo a number of scatterings to reach X-ray energy, and therefore they have forgotten their initial energy and angular distribution. For non-relativistic plasma temperatures the average frequency change on scattering is given by $\langle \Delta x \rangle / x \simeq 4\Theta$ (see Pozdnyakov et al. 1983). This implies, that photons at 5 eV have to scatter 20 times in a plasma with $k_{\rm B}T_{\rm e} = 50\,{\rm keV}$ to achieve an energy of roughly 4 keV.

For disk temperatures higher than e.g. $\Theta = 0.15$ spectral indices about 1 are seen for an optical thickness lower than $2\tau_0 = 1$ for which the anisotropy of the specific intensity becomes important. In particular, our results concerning the degree of anisotropy (which is only weakly dependent on temperature) are relevant to the case of Seyfert galaxies, where plasma temperatures of the order of 100 keV have been suggested (Titarchuk & Mastichiadis 1994; Zdziarski et al. 1995). In these objects, the ratio of optical to X-ray luminosity should be much smaller for objects seen 'edge-on' than for those seen 'face-on' (an effect predicted by Haardt & Maraschi 1993). It may, however, be difficult to disentangle this effect from that of the increased absorption of optical radiation expected in edge-on sources.

It is worth mentioning, that we have used a polarisation averaged treatment of the transport. All current Monte-Carlo codes also use this same approximation, so that the results they obtain are directly comparable to ours.

ACKNOWLEDGEMENTS

We would like to thank A. Mastichiadis for useful discussions and L. Titarchuk for several suggested improvements.

REFERENCES

Ebisawa K., Titarchuk L., Chakrabarti S.K., 1996, Publ. Astron. Soc. Japan 48, 59
Gieseler U.D.J., Kirk J.G., 1996, astro-ph/9612175, A&A in press
Haardt F., Maraschi L., 1993, ApJ 413, 507
Haardt F., Maraschi L., Ghisellini G., 1994, ApJ 432, L95
Hua X.-M., Titarchuk L.G., 1995, ApJ 449, 188
Katz J.I., 1976, ApJ 206, 910
Pomraning G.C., 1973, *Radiation Hydrodynamics*, Pergamon Press, Oxford
Poutanen J., Svensson R., 1996, ApJ 470, 249
Pozdnyakov L.A., Sobol I.M., Sunyaev R.A., 1983, Ap. Space Phys. Rev. 2, 189
Shapiro S.L., Lightman A.P., Eardley D.M., 1976, ApJ 204, 187
Stern B.E., Begelman M.C., Sikora M., Svensson R., 1995a, MNRAS 272, 291
Stern B.E., Poutanen J., Svensson R., Sikora M., Begelman M.C., 1995b, ApJ 449, L13
Sunyaev R.A., Titarchuk L.G., 1980, A&A 86, 121
Sunyaev R.A., Titarchuk L.G., 1985, A&A 143, 374
Sunyaev R.A., Trümper J., 1979, Nature 279, 506
Titarchuk L.G., 1994, ApJ 434, 570
Titarchuk L.G., Lyubarskij Y., 1995, ApJ 450, 876
Titarchuk L.G., Mastichiadis A., 1994, ApJ 433, L33
Zdziarski A.A., 1986, ApJ 303, 94
Zdziarski A.A., Johnson W.N., Done C., Smith D., McNaron-Brown K., 1995, ApJ 438, L63