Particle acceleration at oblique shocks and discontinuities of the density profile

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Abstract

In the theory of diffusive acceleration at oblique shock fronts the question of the existence of a discontinuity of energetic particle density is contentious. The resolution of this problem is interesting both from a theoretical point of view and for the interpretation of observations of particle densities by spacecraft, and high resolution radio observations of the rims of supernova remnants. It can be shown analytically that an isotropic particle distribution at a shock front implies continuity of the particle density — whether or not the shock is oblique. However, if the obliquity of the shock induces an anisotropy, a jump is permitted. We use Monte–Carlo simulations to show that for interesting parameter ranges a jump is indeed produced, with accelerated particles concentrated in a precursor ahead of the shock front.

Controversy — general description

In the test–particle theory of diffusive shock acceleration, the phase space spectral index s of accelerated particles depends solely on the compression ratio r of the shock: $s = 3r/(r-1)$, which results in $s = 4$ for a strong shock (e.g., Axford 1977). This result, like many other analytical predictions (for a review see e.g. Drury 1983), depends on the assumption that the phase space density is close to being isotropic, even at the shock front. In this case, the density profile of accelerated test particles is a continuous function of position (e.g., Kirk 1994, page 262). Moreover, in planar symmetry, the density is constant downstream and drops off exponentially upstream of the shock.

Recently, a discussion has arisen in the literature concerning the occurrence of discontinuities in the density of accelerated particles at an oblique shock front. Whereas Ostrowski (1991) finds a substantial effect, Naito (1995) assert that the density is continuous. Both of these papers present Monte–Carlo simulations of particle acceleration in which the velocity of the shock front is a finite fraction of the speed of light, and take explicit account of a possible anisotropy of the particle distribution. From the theory of diffusion, it is well–known that the anisotropy of particles of speed v is of the order of u/v, where u is the speed at which the shock sweeps through the medium responsible for making the particles diffuse. At an oblique shock front, the speed relevant for particles diffusing along a magnetic field line is $u = u_\mathrm{s}/\cos\Phi$, where u_s is the speed of inflow along the shock normal and Φ is the angle between the shock normal and the magnetic field, measured in the upstream rest frame of the plasma.

The question which we address in this paper is whether or not this anisotropy is associated with a discontinuity in the particle density and under what conditions such an effect could be observed. An approximation which is often used in treating oblique shocks is that in which a particle crossing the shock conserves its magnetic moment (e.g., Decker 1988). Conservation of magnetic moment implies that a particle can be reflected by the magnetic compression at a fast–mode shock front, and the question of the existence of a discontinuity in the particle density is intimately connected with the phenomenon of reflection, as pointed out in an early paper on this subject (Achterberg 1980). If the approximation is adopted, the problem of acceleration at an oblique shock of particles which undergo pitch–angle diffusion along field lines can be solved essentially analytically (Kirk 1989), at least for the case in which the accelerated particles are ultra–relativistic ($v = c$) and build a power–law distribution in momentum. We address the question of the existence of a discontinuity using a new Monte–Carlo code, incorporating the assumption of conservation of magnetic moment (Gieseler 1998a).

The resolution of this question is not only of formal interest, but is also relevant for the interpretation of data taken by the Ulysses spacecraft and of observations of the radio emission of supernova remnants.

Monte–Carlo simulations

We present results from test–particle simulations of accelerated particles at planar shock fronts. The key aspects of the technique have been used and described by several authors (e.g., Kirk 1987; Ostrowski 1991; Baring 1993; Naito 1995), so that a brief description suffices. We consider oblique shocks, where the magnetic field is inclined at an angle Φ with respect to the shock normal in the upstream rest frame, and has no dynamical effect on the plasma flow. The shock speed in this frame is u_s. The gyro-centre of the particle trajectories are followed in the upstream and downstream rest frames of the background plasma. In these frames the momentum $p = |\vec{p}|$ is constant. Particles move along the magnetic field $\vec{B}$ under the influence of small scale irregularities which lead to pitch angle scattering. We use an algorithm for calculating a pitch angle μ_{new} from a given pitch angle μ, which was given by Ostrowski (1991) (see Figure 1 therein). The maximal change of the particle direction is given by $\Delta\Omega_{\text{max}} = 0.1$, which is appropriate for infinitesimal pitch angle scattering ($\Delta\Omega_{\text{max}} = \pi$ corresponds to large angle scattering). The time step Δt for successive scatterings is kept constant. Our simulations make use of the conservation of the adiabatic invariant. Because $|p|$ is conserved in the de Hoffmann/Teller (dHT) frame this leads to $(1 - \mu^2)/B = (1 - (\mu')^2)/B'$ (e.g., Kirk 1994) where a prime denotes downstream quantities. A simulation of an individual particle terminates when a maximum momentum is reached, or when the particle reaches a certain distance from the shock on the downstream side, at which the density distribution has already reached its constant downstream value. These limits vary with u_s and Φ, but they were always chosen such that no boundary effects are important.

We use the dHT frame in order to compare the densities upstream and downstream directly. In this frame the shock is stationary. We normalize the distance x perpendicular to the shock to the dimensionless variable according to $\xi = x\, u_s/(\kappa_\parallel \cos^2\Phi)$, where $\kappa_\parallel$ the parallel diffusion coefficient (e.g., Decker 1988), and $v = c$ is used for the normalisation (in the following $c = 1$).

Figure 1(left) shows the steady state density in the dHT frame with compression ratio $r = 4$, normalised to the value far downstream. The upstream plasma velocity in the dHT frame is $u_s/\cos\Phi = 0.5$. The plot shows the density $n(\xi)$ as a function

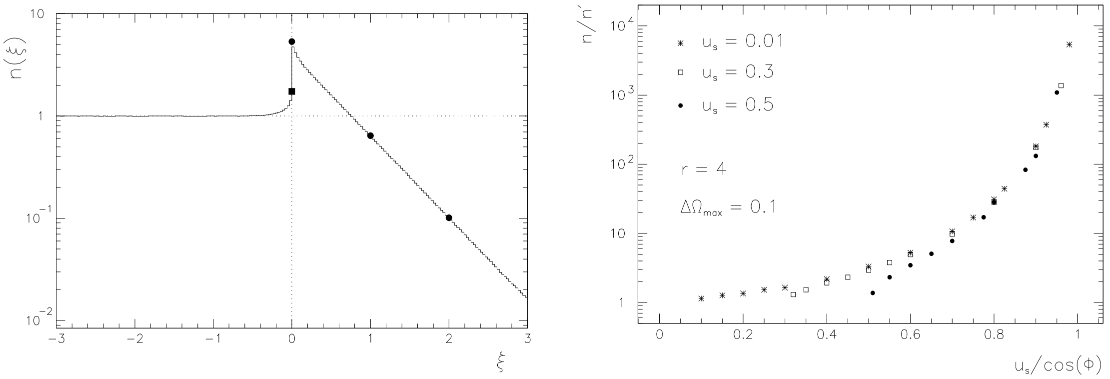

Fig. 1 **Left**: Density profile at an oblique shock with shock speed $u_s = 0.25$ and inclination angle $\Phi = 60°$. The solid line shows a position measurement of particles, whereas the filled dots (square) show a density measured through the flux through a surface with constant distance upstream (downstream) of the shock. (The corresponding spectral index is $s = 3.206 \pm 0.001$). **Right**: Ratio of the upstream to the downstream density at the shock front vs. the intersection velocity of the shock and the magnetic field for infinitesimal pitch angle scattering, and different shock speeds. The values of n/n' are taken from flux measurements at the shock front, indicated by the filled circle and square at $\xi = 0$ in the left Figure.

of the distance to the shock ξ. Because the question of spatial resolution is of crucial importance to our discussion, we use two independent methods to evaluate the density. The solid line shows the contents of spatial 'bins', where particles simply contribute after every time step to the 'bin' at their actual position. An independent way of measuring the density is to count particles which cross a plane at a certain position. The count rate is related to the flux through this plane. To obtain the density, one divides this quantity by the relative velocity of the binned particle and the plane. The three filled dots at $\xi = 0, 1$ and 2 show the density in the upstream region, whereas the filled square shows a density at the downstream side of the shock at $\xi = 0$. Both methods display a discontinuous density profile in Figure 1(left). However, the second method is a more precise measure of the density at $\xi = 0$. For the example of Figure 1(left) we get $n/n' = 3.10 \pm 0.05$, where n represents the filled circle, and n' represents the filled square at $\xi = 0$. Figure 1(right) shows the ratio n/n' for various shock velocities and inclination angles. For $\Phi = 0$ (parallel shock) we get a continuous density ($n = n'$) for all shock speeds, because here no reflection can occur. If the velocity of the intersection point of the shock and the magnetic field exceeds 0.8 of the particle velocity (in the case $v = c$ discussed here), the upstream density becomes more than 10 times the downstream density at the shock front. This ratio increases very rapidly for higher intersection velocities. Our calculations are performed for test particles, but in reality such particles may exert a substantial pressure, so that resolving the structure would be important in calculations which include the back–reaction of the particles on the flow.

However, an increasing magnitude of the maximal change in pitch angle ($\Delta\Omega_{\rm max}$) leads to a breakdown of the highest upstream densities. When the maximal change in pitch angle is determined from $\Delta\Omega_{\rm max} = 1.0$, the ratio n/n' for $r = 4$ do not exceed 30, and for large angle scattering n/n' is always smaller than 2 for all inclination angles which are possible for subluminal oblique shocks (Gieseler 1998b).

Summary and Discussion

We have presented an analysis of particle acceleration at oblique shocks, with special emphasis on the density of accelerated test particles. The corresponding density profile shows a pronounced discontinuity for a large range of parameters. This discontinuity was also found by Ostrowski (1991) (see Figure 6 therein), and our results are quantitatively in very good agreement ($r = 5.28$, $u_s = 0.3$, $\Delta\Omega_{\mathrm{max}} = 0.1$, $u_s/\cos\Phi = 0.6 \Rightarrow n/n' = 6.31 \pm 0.04$; and $u_s/\cos\Phi = 0.8771 \Rightarrow n/n' = 393 \pm 7$). This discontinuity is a result of a reflection of particles hitting the shock from upstream. Especially in the case of small angle scattering, the particles undergo repeated reflections (by which they are accelerated) before they reach a pitch angle at which they can cross the shock into the downstream region. Allowing for a larger maximum value in the change of the pitch angle increases the probability of entering the loss cone, and therefore crossing the shock from upstream to downstream. This reduces the density contrast. For large angle scattering, where the pitch angle is randomised after every scattering event, the pile–up effect is almost absent.

The occurrence of a density jump of accelerated particles at the shock front could be detected in situ by space observations in the solar system. This would be restricted to such cases, where the magnetic field is not too much disturbed, and the scattering law is described through infinitesimal pitch angle scattering. An indirect signature for an pile–up of electrons ahead of a shock front of supernova remnants could be a synchrotron flux, which *de*creases downstream of the shock instead of increasing due to the compressed magnetic field. Whereas this could always happen at a shock which propagates in an inhomogeneous medium, such a signature could be also produced by a shock moving in a homogeneous interstellar medium with an unperturbed oblique magnetic field as a result of a pile–up of reflected particles.

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